# EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

**1.** In a triangle $\angle A = 55°$ and $\angle B = 15°$, then $\frac{c^2 - a^2}{ab}$ is equal to -

(A) 4 (B) 3 (C) 2 (D) 1

**2.** In a triangle ABC a : b : c = $\sqrt{3}$ : 1 : 1, then the triangle is -

(A) right angled triangle (B) obtuse angled triangle

(C) acute angled triangle, which is not isosceles (D) Equilateral triangle

**3.** The sides of a triangle ABC are x, y, $\sqrt{x^2 + y^2 + xy}$ respectively. The size of the greatest angle in radians is -

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) none of these

**4.** In a $\Delta ABC\left(\frac{a^2}{\sin A}+\frac{b^2}{\sin B}+\frac{c^2}{\sin C}\right)$. $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ simplifies to -

(A) $2\Delta$ (B) $\Delta$ (C) $\frac{\Delta}{2}$ (D) $\frac{\Delta}{4}$

(where $\Delta$ is the area of triangle)

**5.** If $p_1$, $p_2$, $p_3$ are the altitudes of a triangle from its vertices A, B, C and $\Delta$, the area of the triangle ABC, then $\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3}$ is equal to -

(A) $\frac{s}{\Delta}$ (B) $\frac{s-c}{\Delta}$ (C) $\frac{s-b}{\Delta}$ (D) $\frac{s-a}{\Delta}$

**6.** If in a triangle ABC angle B = 90 then $\tan^2 A/2$ is -

(A) $\frac{b-c}{a}$ (B) $\frac{b-c}{b+c}$ (C) $\frac{b+c}{b-c}$ (D) $\frac{b+c}{a}$

**7.** In a triangle ABC, if $\frac{a^3+b^3+c^3}{\sin^3 A+\sin^3 B+\sin^3 C}=343$, the diameter of the circle circumscribing the triangle is -

(A) 7 units (B) 14 units (C) 21 units (D) none of these

**8.** In a $\Delta ABC$ if b + c = 3a then $\cot\frac{B}{2}.\cot\frac{C}{2}$ has the value equal to -

(A) 4 (B) 3 (C) 2 (D) 1

**9.** If $\frac{a}{\sin A}$ = K, then the area of $\Delta ABC$ in terms of K and sines of the angles is -

(A) $\frac{K^2}{4}\sin A\sin B\sin C$ (B) $\frac{K^2}{2}\sin A\sin B\sin C$

(C) $2K^2\sin A\sin B\sin(A + B)$ (D) none

**10.** In a $\Delta$ABC, $\angle C = 60°$ & $\angle A = 75°$. If D is a point on AC such that the area of the $\Delta$BAD is $\sqrt{3}$ times the area of the $\Delta$BCD, then the $\angle ABD =$

(A) 60 (B) 30 (C) 90 (D) none of these

**11.** In a $\Delta ABC$, a semicircle is inscribed, whose diameter lies on the side c. Then the radius of the semicircle is (Where $\Delta$ is the area of the triangle ABC)

(A) $\frac{2\Delta}{a+b}$ (B) $\frac{2\Delta}{a+b-c}$ (C) $\frac{2\Delta}{s}$ (D) $\frac{c}{2}$

**12.** In a triangle ABC, right angled at B, the inradius is -

(A) $\frac{AB+BC-AC}{2}$ (B) $\frac{AB+AC-BC}{2}$ (C) $\frac{AB+BC+AC}{2}$ (D) none

**13.** In triangle ABC where A, B, C are acute, the distances of the orthocentre from the sides are in the proportion

(A) cos A : cos B : cos C (B) sin A : sin B : sin C

(C) sec A : sec B : sec C (D) tan A : tan B : tan C

**14.** In a $\Delta ABC$, the value of $\frac{a\cos A+b\cos B+c\cos C}{a+b+c}$ is equal to -

(A) $\frac{r}{R}$ (B) $\frac{R}{2r}$ (C) $\frac{R}{r}$ (D) $\frac{2r}{R}$

**15.** If the orthocentre and circumcentre of a triangle ABC be at equal distances from the side BC and lie on the same side of BC then $\tan B \tan C$ has the value equal to -

(A) 3 (B) $\frac{1}{3}$ (C) $-3$ (D) $-\frac{1}{3}$

**16.** In an equilateral triangle, inradius r, circumradius R & ex-radius $r_1$ are in -

(A) A.P. (B) G.P. (C) H.P. (D) none of these

**17.** With usual notation in a $\Delta ABC$ $\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left(\frac{1}{r_2}+\frac{1}{r_3}\right)\left(\frac{1}{r_3}+\frac{1}{r_1}\right)=\frac{K\ R^3}{a^2\ b^2\ c^2}$ then K has value equal to -

(A) 1 (B) 16 (C) 64 (D) 128

**18.** In a triangle ABC, $\frac{r_1+r_2}{1+\cos C}$ is equal to -

(A) $2ab/c\Delta$ (B) $(a+b)/c\Delta$ (C) $abc/2\Delta$ (D) $abc/\Delta^2$

**19.** With usual notations in a triangle ABC, if $r_1 = 2r_2 = 2r_3$ then -

(A) $4a = 3b$ (B) $3a = 2b$ (C) $4b = 3a$ (D) $2a = 3b$

**20.** If $r_1$, $r_2$, and $r_3$ be the radii of excircles of the triangle ABC, then $\frac{\sum r_1}{\sqrt{\sum r_1 r_2}}$ is equal to -

(A) $\sum \cot\frac{A}{2}$ (B) $\sum \cot\frac{A}{2}\cot\frac{B}{2}$ (C) $\sum \tan\frac{A}{2}$ (D) $\prod \tan\frac{A}{2}$

**21.** If in a triangle PQR, sin P, sin Q, sin R are in A.P., then - **[JEE 98]**

(A) the altitudes are in A.P. (B) the altitudes are in H.P.

(C) the medians are in G.P. (D) the medians are in A.P.

**22.** In $\Delta ABC$, if $r : r_1 : R = 2 : 12 : 5$, where all symbols have their usual meaning, then -

(A) $\Delta ABC$ is an acute angled triangle (B) $\Delta ABC$ is an obtuse angled triangle

(C) $\Delta ABC$ is right angled which is not isosceles (D) $\Delta ABC$ is isosceles which is not right angled

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

**23.** In a $\Delta ABC$, $A = \frac{\pi}{3}$ and b : c = 2 : 3. If $\tan\alpha = \frac{\sqrt{3}}{5}, 0 < \alpha < \frac{\pi}{2}$, then -

(A) $B = 60° + \alpha$ (B) $C = 60° + \alpha$ (C) $B = 60° - \alpha$ (D) $C = 60° - \alpha$

**24.** In a triangle ABC, points D and E are taken on sides BC such that DB = DE = EC. If $\angle ADE = \angle AED = \theta$, then -

(A) $\tan\theta = 3\tan B$ (B) $\tan\theta = 3\tan C$ (C) $\tan A = \frac{6\tan\theta}{\tan^2\theta - 9}$ (D) $9\cot^2\frac{A}{2} = \tan^2\theta$

**25.** If a, b, A are given in a triangle and $c_1$ and $c_2$ are two possible values of third side such that $c_1^2 + c_1c_2 + c_2^2 = a^2$, then A is equal to -

(A) 30 (B) 60 (C) 90 (D) 120

**26.** In a $\Delta ABC$, AD is the bisector of the angle A meeting BC at D. If I is the incentre of the triangle, then AI : DI is equal to -

(A) (sinB + sinC) : sinA (B) (cosB + cosC) : cosA

(C) $\cos\left(\frac{B-C}{2}\right) : \cos\left(\frac{B+C}{2}\right)$ (D) $\sin\left(\frac{B-C}{2}\right) : \sin\left(\frac{B+C}{2}\right)$



CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | D | B | A | B | B | B | A | C | B | B |
| Que. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Ans. | A | A | C | A | A | A | C | C | C | C |
| Que. | 21 | 22 | 23 | 24 | 25 | 26 | | | | |
| Ans. | B | C | B,C | A,B,C,D | B | A,C | | | | |

# EXERCISE - 02

**BRAIN TEASERS**

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If A, B, C are angles of a triangle which of the following will not imply it is equilateral -
(A) $\tan A + \tan B + \tan C = 3\sqrt{3}$ (B) $\cot A + \cot B + \cot C = \sqrt{3}$
(C) $a + b + c = 2R$ (D) $a^2 + b^2 + c^2 = 9R^2$

2. In a $\Delta ABC$, $\frac{s}{R}$ is equal to -
(A) $\sin A + \sin B + \sin C$ (B) $4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ (C) $4\sin A\sin B\sin C$ (D) $\frac{\Delta s}{abc}$

3. If $\cos A + \cos B + 2\cos C = 2$ then the sides of the $\Delta ABC$ are in-
(A) A.P. (B) G.P. (C) H.P. (D) none

4. The line $\frac{x}{6}+\frac{y}{8}=1$ cuts the co-ordinate axis at A & B. If O is origin, then $\prod\sin\frac{A}{2}$ for the triangle OAB is -
(A) 5/6 (B) 1/10 (C) 5/4 (D) none of above

5. In a triangle ABC, CD is the bisector of the angle C. If $\cos\frac{C}{2}$ has the value $\frac{1}{3}$ and $\ell(CD) = 6$, then $\left(\frac{1}{a}+\frac{1}{b}\right)$ has the value equal to -
(A) $\frac{1}{9}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) none

6. In the triangle ABC, CH and CM are the lengths of the altitude and median to the base AB. If a = 10, b = 26, c = 32 then length HM is -
(A) 5 (B) 7 (C) 9 (D) none

7. D, E, F are the foot of the perpendiculars from vertices A, B, C to sides BC, CA, AB respectively, and H is the orthocentre of acute angled triangle ABC; where a, b, c are the sides of triangle ABC, then
(A) H is the incentre of triangle DEF
(B) A, B, C are excentres of triangle DEF
(C) Perimeter of $\Delta DEF$ is $a\cos A + b\cos B + c\cos C$
(D) Circumradius of triangle DEF is $\frac{R}{2}$, where R is circumradius of $\Delta ABC$.

8. If x, y and z are the distances of incentre from the vertices of the triangle ABC respectively then $\frac{abc}{xyz}$ is equal to -
(A) $\prod\tan\frac{A}{2}$ (B) $\sum\cot\frac{A}{2}$ (C) $\sum\tan\frac{A}{2}$ (D) $\prod\cot\frac{A}{2}$

9. The medians of a $\Delta ABC$ are 9 cm, 12 cm and 15 cm respectively. Then the area of the triangle is -
(A) 96 sq cm (B) 84 sq cm (C) 72 sq cm (D) 60 sq cm

10. In an isosceles $\Delta ABC$, if the altitudes intersect on the inscribed circle then the cosine of the vertical angle 'A' is :
(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) none

**11.** In triangle ABC, cosA + 2cosB + cosC = 2, then -

(A) $\tan\frac{A}{2}\tan\frac{C}{2}=3$ (B) $\cot\frac{A}{2}\cot\frac{C}{2}=3$ (C) $\cot\frac{A}{2}+\cot\frac{C}{2}=2\cot\frac{B}{2}$ (D) $\tan\frac{A}{2}\tan\frac{C}{2}=0$

**12.** If in a triangle ABC p, q, r are the altitudes from the vertices A, B, C to the opposite sides, then which of the following does not hold good ?

(A) $(\Sigma p)\left(\Sigma\frac{1}{p}\right) = (\Sigma a)\left(\Sigma\frac{1}{a}\right)$ (B) $(\Sigma p)\ (\Sigma a) = \left(\Sigma\frac{1}{p}\right)\left(\Sigma\frac{1}{a}\right)$

(C) $(\Sigma p)\ (\Sigma pq)\ (\Pi a) = (\Sigma a)\ (\Sigma ab)\ (\Pi p)$ (D) $\left(\Sigma\frac{1}{p}\right)\Pi\left(\frac{1}{p}+\frac{1}{q}-\frac{1}{r}\right)\Pi a^2 = 16R^2$

**13.** AD, BE and CF are the perpendiculars from the angular points of a $\Delta ABC$ upon the opposite sides. The perimeters of the $\Delta DEF$ and $\Delta ABC$ are in the ratio -

(A) $\frac{2r}{R}$ (B) $\frac{r}{2R}$ (C) $\frac{r}{R}$ (D) $\frac{r}{3R}$

Where r is the inradius and R is circum-radius of the $\Delta ABC$

**14.** If 'O' is the circum centre of the $\Delta ABC$ and $R_1$, $R_2$ and $R_3$ are the radii of the circumcircles of triangles OBC, OCA and OAB respectively then $\frac{a}{R_1}+\frac{b}{R_2}+\frac{c}{R_3}$ has the value equal to -

(A) $\frac{abc}{2R^3}$ (B) $\frac{R^3}{abc}$ (C) $\frac{4\Delta}{R^2}$ (D) $\frac{abc}{R^3}$

**15.** In a triangle ABC, $(r_1 - r)\ (r_2 - r)\ (r_3 - r)$ is equal to -

(A) $4Rr^2$ (B) $\frac{4abc.\Delta}{(a+b+c)^2}$

(C) $16R^3(\cos A + \cos B + \cos C - 1)$ (D) $r^3 \operatorname{cosec}\frac{A}{2}\operatorname{cosec}\frac{B}{2}\operatorname{cosec}\frac{C}{2}$

**16.** Two rays emanate from the point A and form an angle of 43 with one another. Lines $L_1$, $L_2$ and $L_3$ (no two of which are parallel) each form an isosceles triangle with the original rays. The largest angle of the triangle formed by lines $L_1$, $L_2$ and $L_3$ is -

(A) 127 (B) 129 (C) 133 (D) 137

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | C | A,B | A | B | A | C | A,B,C,D | B,D | C | A |
| Que. | 11 | 12 | 13 | 14 | 15 | 16 | | | | |
| Ans. | B,C | B | C | C,D | A,B,D | B | | | | |

# EXERCISE - 03

## MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. If external angle bisector of any angle of triangle ABC is parallel to the opposite base then triangle is isosceles.
2. Sides of the pedal triangle of any acute or obtuse angle triangle are given by Rsin2A, Rsin2B and Rsin2C.
3. In the triangle ABC, the altitudes $p_1$, $p_2$, $p_3$ are in AP, then a, b, c are in HP.
4. In a triangle ABC, if $a^4 - 2(b^2 + c^2)a^2 + b^4 + b^2c^2 + c^4 = 0$, then $\angle A$ is 60 or 120

FILL IN THE BLANKS

1. In a $\Delta ABC$, tan A : tan B : tan C = 1 : 2 : 3. Hence sinA : sinB : sinC = ____________.
2. In triangle ABC, if a = 2, b = 3 and tan A = $\sqrt{\frac{3}{5}}$ then the two possible values of the side c are $K_1\sqrt{10}$ and $K_2\sqrt{10}$ then $K_1$ and $K_2$ are equal to ____________ and ______________.
3. If f, g and h are the lengths of the perpendiculars from the circumcentre on the sides a, b and c of a triangle ABC respectively then $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = K\frac{abc}{fgh}$ where K has the value equal to _______________.

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1. If $p_1$, $p_2$, $p_3$ are altitudes of a triangle ABC from the vertices A, B, C respectively and $\Delta$ is the area of the triangle and s is semi perimeter of the triangle, then match the columns

| | Column-I | | Column-I |
|---|---|---|---|
| (A) | If $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{2}$ then the least value of $p_1p_2p_3$ is | (p) | $\frac{1}{R}$ |
| (B) | The value of $\frac{\cos A}{p_1}+\frac{\cos B}{p_2}+\frac{\cos C}{p_3}$ is | (q) | 216 |
| (C) | The minimum value of $\frac{b^2p_1}{c}+\frac{c^2p_2}{a}+\frac{a^2p_3}{b}$ is | (r) | $6\Delta$ |
| (D) | The value of $p_1^{-2} + p_2^{-2} + p_3^{-2}$ is | (s) | $\frac{\Sigma a^2}{4\Delta^2}$ |

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : If two sides of a triangle are 4 and 5, then its area lies in (0, 10]

**Because** :

**Statement-II** : Area of a triangle $=\frac{1}{2}$ ab sinC and sinC $\in$ (0, 1]

(A) A (B) B (C) C (D) D

2. **Statement-I** : Perimeter of a regular pentagon inscribed in a circle with centre O and radius a cm equals $10 a \sin 36$ cm

**Because** :

**Statement-II** : Perimeter of a regular polygon inscribed in a circle with centre O and radius a cm equals $(3n - 5)\ \sin\left(\frac{360°}{2n}\right)$ cm, then it is n sided, where $n \geq 3$

(A) A (B) B (C) C (D) D

3. **Statement-I** : The statement that circumradius and inradius of a triangle are 12 and 8 respectively can not be correct.

**Because** :

**Statement-II** : Circumradius $\geq$ 2 (inradius)

(A) A (B) B (C) C (D) D

4. **Statement-I** : In any triangle ABC, the minimum value of $\frac{r_1 + r_2 + r_3}{r}$ is 9

**Because** :

**Statement-II** : For any three numbers AM $\geq$ GM

(A) A (B) B (C) C (D) D

5. **Statement-I** : Area of triangle having sides greater than 9 can be smaller than area of triangle having sides less than 3.

**Because** :

**Statement-II** : Sine of an angle of triangle can take any value in (0, 1]

(A) A (B) B (C) C (D) D

## COMPREHENSION BASED QUESTIONS

### Comprehension # 1

Let $A_n$ be the area that is outside a n-sided regular polygon and inside it's circumscribing circle. Also $B_n$ is the area inside the polygon and outside the circle inscribed in the polygon. Let R be the radius of the circle circumscribing n-sided polygon.

**On the basis of above information, answer the following questions :**

1. If n = 6 then $A_n$ is equal to-

(A) $R^2\left(\frac{\pi - \sqrt{3}}{2}\right)$ (B) $R^2\left(\frac{2\pi - 6\sqrt{3}}{2}\right)$ (C) $R^2\left(\pi - \sqrt{3}\right)$ (D) $R^2\left(\frac{2\pi - 3\sqrt{3}}{2}\right)$

2. If n = 4 then $B_n$ is equal to -

(A) $R^2\frac{(4-\pi)}{2}$ (B) $R^2\frac{(4-\pi\sqrt{2})}{2}$ (C) $R^2\frac{(4\sqrt{2}-\pi)}{2}$ (D) none of these

3. $\frac{A_n}{B_n}$ is equal to $\left(\theta=\frac{\pi}{n}\right)$ -

(A) $\frac{2\theta-\sin 2\theta}{\sin 2\theta-\theta\cos^2\theta}$ (B) $\frac{2\theta-\sin\theta}{\sin 2\theta-\theta\cos^2\theta}$ (C) $\frac{\theta-\cos\theta\sin\theta}{\cos\theta(\sin\theta-\theta\cos\theta)}$ (D) none of these

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE-3

- *True / False*

1. T 2. F 3. T 4. T

- *Fill in the Blanks*

1. $\sqrt{5} : 2\sqrt{2} : 3$ 2. 1 and 1/2 3. $\frac{1}{4}$

- *Match the Column*

1. (A) → (q), (B) → (p), (C) → (r), (D) → (s)

- *Assertion & Reason*

1. A 2. C 3. A 4. C 5. A

- *Comprehension Based Questions*

Comprehension # 1 : 1. D 2. A 3. C

# EXERCISE - 04 [A]

## CONCEPTUAL SUBJECTIVE EXERCISE

1. Prove that : $4R \sin A \sin B \sin C = a\cos A + b\cos B + c\cos C$.

2. Prove that : $a\cos B\cos C + b\cos C\cos A + c\cos A\cos B = \frac{\Delta}{R}$

3. If $p_1, p_2, p_3$ are the altitudes of a triangle from the vertices A, B, C & $\Delta$ denotes the area of the triangle, prove that $\frac{1}{p_1}+\frac{1}{p_2}-\frac{1}{p_3}=\frac{2ab}{(a+b+c)\Delta}\cos^2\frac{C}{2}$.

4. Prove that : $\frac{abc}{s}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=\Delta$

5. For any triangle ABC, if B = 3C, show that $\cos C = \sqrt{\frac{b+c}{4c}}$ & $\sin\frac{A}{2}=\frac{b-c}{2c}$.

6. ABC is a triangle. D is the mid point of BC. If AD is perpendicular to AC, then prove that $\cos A \cdot \cos C = \frac{2(c^2-a^2)}{3ac}$.

7. Let $1 < m < 3$. In a triangle ABC, if $2b = (m+1)a$ & $\cos A = \frac{1}{2}\sqrt{\frac{(m-1)(m+3)}{m}}$ prove that there are two values to the third side, one of which is m times the other.

8. Prove that : $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}$

9. Prove that : $r_1 + r_2 + r_3 - r = 4R$

10. Prove that : $\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3}=0$

11. Consider a $\Delta$DEF, the pedal triangle of the $\Delta$ABC such that A–F–B and B–D–C are collinear. If H is the incentre of $\Delta$DEF and $R_1$, $R_2$, $R_3$ are the circumradii of the quadarilaterals AFHE; BDHF and CEHD respectively, then prove that $\sum R_1 = R + r$ where R is the circumradius and r is the inradius of $\Delta$ABC.

12. DEF is the triangle formed by joining the points of contact of the incircle with the sides of the triangle ABC, prove that

(a) its sides are $2r\cos\frac{A}{2}$, $2r\cos\frac{B}{2}$ and $2r\cos\frac{C}{2}$ where r is the radius of incircle of $\Delta$ABC.

(b) its angles are $\frac{\pi}{2}-\frac{A}{2}$, $\frac{\pi}{2}-\frac{B}{2}$ and $\frac{\pi}{2}-\frac{C}{2}$

(c) its area is $\frac{r^2 s}{2R}$ where 's' is the semiperimeter and R is the circumradius of the $\Delta$ABC.

# EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. If sides a, b, c, of the triangle ABC are in A.P., then prove that

$\sin^2\frac{A}{2}\text{cosec}\,2A;\ \sin^2\frac{B}{2}\text{cosec}\,2B\ ;\ \sin^2\frac{C}{2}\text{cosec}\,2C$ are in H.P.

2. Sides a, b, c of the triangle ABC are in H.P., then prove that
cosecA (cosecA + cot A) ; cosec B (cosecB + cotB) & cosecC (cosecC + cot C) are in A.P.

3. In a $\Delta$ ABC, GA,GB,GC makes angles $\alpha, \beta, \gamma$ with each other where G is the centroid to the $\Delta$ ABC then show that, cotA + cotB + cotC + cot$\alpha$ + cot $\beta$ +cot$\gamma$ =0.

4. In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ & it divides the angle A into angles of 30 & 45 . Find the length of the side BC.

5. Prove that : $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}=\frac{1}{r}-\frac{1}{2R}$

6. Prove that : $\frac{\tan\frac{A}{2}}{(a-b)(a-c)}+\frac{\tan\frac{B}{2}}{(b-a)(b-c)}+\frac{\tan\frac{C}{2}}{(c-a)(c-b)}=\frac{1}{\Delta}$

7. Prove that in a triangle $\frac{bc}{r_1}+\frac{ca}{r_2}+\frac{ab}{r_3}=2R\left[\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)-3\right]$.

8. In a triangle the angles A, B, C are in A.P. Show that $2\cos\frac{A-C}{2}=\frac{a+c}{\sqrt{a^2-ac+c^2}}$.

9. In a scalene triangle ABC the altitudes AD & CF are dropped form the vertices A & C to the sides BC & AB. The area of $\Delta$ABC is known to be equal to 18, the area of triangle BDF is equal to 2 and length of segment DF is equal to $2\sqrt{2}$ . Find the radius of the circle circumscribing $\Delta$ABC.

10. With reference to a given circle, $A_1$ and $B_1$ are the areas of the inscribed and circumscribed regular polygons of n sides, $A_2$ and $B_2$ are corresponding quantities for regular polygons of 2n sides : Prove that
(a) $A_2$ is a geometric mean between $A_1$ and $B_1$
(b) $B_2$ is a harmonic mean between $A_2$ and $B_1$

11. Let a, b, c be the sides of a trianlge & $\Delta$ its area. Prove that $a^2+b^2+c^2\geq 4\ \sqrt{3}\Delta$ , and find when does the equality hold ?

12. If in a triangle of base ‘a’, the ratio of the other two sides is r ( < 1) . Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$.

13. If the bisector of angle C of triangle ABC meets AB in D & the circumcircle in E prove that , $\frac{CE}{DE}=\frac{(a+b)^2}{c^2}$.

| BRAIN STORMING SUBJECTIVE EXERCISE | ANSWER KEY | EXERCISE-4(B) |
|---|---|---|
| 4. a = 2 | 9. 9/2 units | 11. b = c & A = 60 |

# EXERCISE - 05

## JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

**1.** If $\Delta$ is the area of a triangle with side lengths a, b, c, then show that: $\Delta \le \frac{1}{4}\sqrt{(a+b+c)abc}$

Also show that equality occurs in the above inequality if and only if a = b = c. [JEE 2001]

**2.** Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC(R being the radius of the circumcircle)?

(A) a, sinA, sinB (B) a, b, c (C) a, sinB, R (D) a, sinA, R

[JEE 2002 (Scr), 3]

**3.** If $I_n$ is the area of n sided regular polygon inscribed in a circle of unit radius and $O_n$ be the area of the polygon circumscribing the given circle, prove that $I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$

[JEE 2003, Mains, 4 out of 60]

**4.** The ratio of the sides of a triangle ABC is $1 : \sqrt{3} : 2$. The ratio A : B : C is [JEE 2004 (Screening)]

(A) 3 : 5 : 2 (B) $1 : \sqrt{3} : 2$ (C) 3 : 2 : 1 (D) 1 : 2 : 3

**5.** (a) In $\Delta$ABC, a, b, c are the lengths of its sides and A, B, C are the angles of triangle ABC. The correct relation is [JEE 2005 (Screening)]

(A) $(b-c)\sin\left(\frac{B-C}{2}\right) = a\cos\left(\frac{A}{2}\right)$ (B) $(b-c)\cos\left(\frac{A}{2}\right) = a\sin\left(\frac{B-C}{2}\right)$

(C) $(b+c)\sin\left(\frac{B+C}{2}\right) = a\cos\left(\frac{A}{2}\right)$ (D) $(b-c)\cos\left(\frac{A}{2}\right) = 2a\sin\left(\frac{B+C}{2}\right)$

(b) Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the points of contact. [JEE 2005 (Mains), 2]

**6.** (a) Given an isosceles triangle, whose one angle is 120 and radius of its incircle is $\sqrt{3}$. Then the area of triangle in sq. units is [JEE 2006, 3]

(A) $7 + 12\sqrt{3}$ (B) $12 - 7\sqrt{3}$ (C) $12 + 7\sqrt{3}$ (D) $4\pi$

(b) Internal bisector of $\angle$A of a triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of $\Delta$ABC then

(A) AE is HM of b and c (B) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$

(C) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$ (D) the triangle AEF is isosceles [JEE 2006, 5]

**7.** Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30 . The absolute value of the difference between the areas of these triangles is

[JEE 2009, 5]

8. (a) If the angle A,B and C of a triangle are in an arithmetic progression and if a,b and c denote the length of the sides opposite to A,B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$, is -

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

(b) Consider a triangle ABC and let a,b and c denote the length of the sides opposite to vertices A,B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then $r^2$ is equal to

(c) Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a,b and c denote the lengths of the sides opposite to A,B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is/are [JEE 2010, 3+3+3]

(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

9. Let PQR be a triangle of area $\Delta$ with a = 2, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

[JEE 2012, 3M, –1M]

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

| PREVIOUS YEARS QUESTIONS | ANSWER KEY | EXERCISE-5 |
|---|---|---|

2. D 4. D 5. (a) B; (b) $\sqrt{5}$ 6. (a) C, (b) A, B, C, D 7. 4
8. (a) D, (b) 3, (c) B 9. C